* 0019655321201 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 2 8 9 3 0 0 5 2 2 8 *

FURTHER MATHEMATICS **9231/13**

Paper 1 Further Pure Mathematics 1 **May/June 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} k & 1 & 0 \\ 6 & 5 & 2 \\ -1 & 3 & -k \end{pmatrix},$$

where k is a real constant.

(a) Show that $\mathbf{A}$ is non-singular. [3]

(b) Given that $\mathbf{A}^{-1} = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 0 & 0 \\ -\frac{23}{2} & \frac{1}{2} & 3 \end{pmatrix}$, find the value of k. [2]

2 The cubic equation $x^3 + 2x^2 + 3x + 1 = 0$ has roots α, β, γ.

(a) Find a cubic equation whose roots are $\alpha^2 + 1$, $\beta^2 + 1$, $\gamma^2 + 1$. [3]

(b) Find the value of $(\alpha^2+1)^2+(\beta^2+1)^2+(\gamma^2+1)^2$. [2]

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(c) Find the value of $(\alpha^2+1)^3+(\beta^2+1)^3+(\gamma^2+1)^3$. [2]

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3 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 7 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix **M** represents a sequence of two geometrical transformations in the *x*-*y* plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

(b) Find the equations of the invariant lines, through the origin, of the transformation represented by **M**. [5]

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The triangle *DEF* in the *x*-*y* plane is transformed by **M** onto triangle *PQR*.

(c) Given that the area of triangle *PQR* is $35\,\text{cm}^2$, find the area of triangle *DEF*. [2]

4 **(a)** Prove by mathematical induction that, for all positive integers n,

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1).$$

[5]

The sum S_n is defined by $S_n = \sum_{r=1}^{n} r^4$.

(b) Using the identity

$$(2r+1)^5 - (2r-1)^5 \equiv 160r^4 + 80r^2 + 2,$$

show that $S_n = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$. [6]

(c) Find the value of $\lim_{n \to \infty}\left(n^{-5}S_n\right)$. [2]

5 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} - 2\mathbf{k})$ and $\mathbf{r} = -3\mathbf{i} + 4\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

(a) Find the shortest distance between l_1 and l_2. [5]

The plane Π_1 contains l_1 and is parallel to l_2.

(b) Obtain an equation of Π_1 in the form $px + qy + rz = s$. [2]

(c) The point $(1, 1, 1)$ lies on the plane Π_2.

It is given that the line of intersection of the planes Π_1 and Π_2 passes through the point $(0, 0, 2)$ and is parallel to the vector $\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$.

Obtain an equation of Π_2 in the form $ax + by + cz = d$. [3]

6 The curve C has equation $y = \frac{x+1}{x^2+3}$.

(a) Show that C has no vertical asymptotes and state the equation of the horizontal asymptote. [2]

(b) Find the coordinates of any stationary points on C. [4]

(c) Sketch C, stating the coordinates of the intersections with the axes. [3]

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(d) Sketch $y^2 = \dfrac{x+1}{x^2+3}$, stating the coordinates of the stationary points and the intersections with the axes. [4]

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7 The curve C has polar equation $r^2 = \sin 2\theta \cos\theta$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the equation of the line of symmetry. [3]

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(b) Find a Cartesian equation for C. [3]

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(c) Find the total area enclosed by C. [4]

(d) Find the greatest distance of a point on C from the pole. [6]

Additional page

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