



Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 20 pages. Any blank pages are indicated.

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The matrix A is given by

$$\mathbf{A} = \begin{pmatrix} k & 1 & 0 \\ 6 & 5 & 2 \\ -1 & 3 & -k \end{pmatrix},$$

3

where k is a real constant.

Given that $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, find the	value of k.
Given that $\mathbf{A}^{-1} = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 0 & 0 \\ -\frac{23}{2} & \frac{1}{2} & 3 \end{pmatrix}$, find the	
$\left(\begin{array}{ccc} -\frac{23}{2} & \frac{1}{2} & 3 \end{array}\right)^{\frac{1}{2}}$	
$\left(-\frac{23}{2} \frac{1}{2} 3\right)$	

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(c)

(b)

Find the value of $(\alpha^2 + 1)^2 + (\beta^2 + 1)^2 + (\gamma^2 + 1)^2$.	[2]
Find the value of $(\alpha^2 + 1)^3 + (\beta^2 + 1)^3 + (\gamma^2 + 1)^3$.	[2]
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- **3** The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 7 & 0 \\ 0 & 1 \end{pmatrix}$.
 - (a) The matrix M represents a sequence of two geometrical transformations in the x-y plane.

Give full details of each transformation, and make clear the order in which they are applied.	[4]
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(b) Find the equations of the invariant lines, through the origin, of the transformation represented by M. [5]

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The	triangle DEF in the x-y plane is transformed by M onto triangle PQR .	
(c)	Given that the area of triangle PQR is $35 \mathrm{cm}^2$, find the area of triangle DEF .	
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4 (a)	Prove by	y mathematical	induction tha	t, for all	positive :	integers n,
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$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1).$	[5]
r=1	

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The sum S_n is defined by $S_n = \sum_{r=1}^n r^4$.

(b) Using the identity

$(2r+1)^5 - (2r-1)^5 \equiv 160r^4 + 80r^2 + 2,$	
show that $S_n = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$.	[6]
Find the value of $\lim_{n \to \infty} \left(n^{-5} S_n \right)$.	[2]

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(c)

- 5 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 4\mathbf{j} \mathbf{k} + \lambda(\mathbf{j} 2\mathbf{k})$ and $\mathbf{r} = -3\mathbf{i} + 4\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.
 - (a) Find the shortest distance between l_1 and l_2 . [5]

The plane Π_1 contains l_1 and is parallel to l_2 .

(b) Obtain an equation of Π_1 in the form px + qy + rz = s. [2]



(c) The point (1,1,1) lies on the plane Π_2 .

It is given that the line of intersection of the planes Π_1 and Π_2 passes through the point $(0,0,2)$ and is parallel to the vector $\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$.			
Obtain an equation of Π_2 in the form $ax + by + cz = d$. [3]			

The curve C has equation $y = \frac{x+1}{x^2+3}$

(b) Find the coordinates of any stationary points on *C*.

(a)	Show that C has no vertical asymptotes and state the equation of the horizontal asymptote.

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[4]



(c) Sketch *C*, stating the coordinates of the intersections with the axes.

[3]

(d) Sketch $y^2 = \frac{x+1}{x^2+3}$, stating the coordinates of the stationary points and the intersections with the axes. [4]



- 7 The curve C has polar equation $r^2 = \sin 2\theta \cos \theta$, for $0 \le \theta \le \pi$.
 - (a) Sketch C and state the equation of the line of symmetry.

[3]

(b)	Find a Cartesian equation for <i>C</i> .	[3]

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Find the total area enclosed by <i>C</i> .	15	[4
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Find the greatest distance of a point on C from the pole.	[6]
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17 Additional page

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